Review - Chapter 4

1. Use transformations to graph \( f(x) = 3^x + 2 \). Determine the domain, range and find the equation of the horizontal asymptote. Plot at least 3 points on the graph of the basic function and use them to perform transformations. Do one transformation at a time and write the equation for each function.

2. The graph of \( y = e^x \) is reflected about the y-axis and then shifted three units to the right. Write the equation of the function in the final position.

3. Use transformations to graph \( f(x) = -2^{x+4} - 3 \). Determine the domain, range and find the equation of the horizontal asymptote. Plot at least 3 points on the graph of the basic function and use them to perform transformations. Do one transformation at a time and write the equation for each function.

4. Express as a single logarithm. Assume all variables are positive and \( b \neq 1 \)
   
   \[
   a) \quad 2\log_b t - \frac{4}{3}\log_b s + \frac{1}{3}\log_b v - 4\log_b u \\
   b) \quad 2\ln x - 3\ln y - 4\ln z \\
   c) \quad \log\left(\frac{p}{q}\right) + \log\left(\frac{q}{s}\right) - \log\left(\frac{p}{s^2}\right)
   
5. Approximate \( \log_5 (7) \) to 3 decimal places. (use the change of the base formula)

6. Use the properties of logarithms to find the exact values of the expressions. Do not use a calculator.
   
   \[
   a) \quad \log_4 24 - \log_4 6 \\
   b) \quad 10^{\log_{30} - \log_5} \\
   c) \quad \log_3 30 \cdot \log_{30} 9
   \]
7. Solve the given equation. Give exact solutions. Do not use a calculator.
   a) \(2^{x^2-3} = 64\)
   b) \(e^{x-2} = \left(\frac{1}{e^2}\right)^{x-1}\)
   c) \(e^{2x-1} = 2\)
   d) \(\log(3 + x) - \log(x-5) = \log 5\)
   e) \(\log_2(3x-2) - \log_2(x-5) = 4\)
   f) \(3^x = 9^{x-1} \cdot 27^{1-3x}\)
   g) \(\log(x^2 + 5x + 16) = 1\)
   h) \(3 \cdot 2^{2x-1} + 5 = 14\)

8. Let \(f(x) = 2^x\). Describe a sequence of transformations that results in \(g(x) = 3 \cdot 2^{2x-1} + 4\)

9. Find the exact value of the logarithmic expression. Do not use a calculator.
   a) \(\log_2\left(\frac{1}{27}\right)\)
   b) \(\log_2\left(\frac{1}{\sqrt{5}}\right)\)

10. Find the domain of
    a) \(f(x) = \log\left(\frac{x+2}{3-x}\right)\)
    b) \(f(x) = \log_2(2x - x^3)\)
    c) \(f(x) = \log\left(x^4 + x^3 - x^2 + x - 2\right)\)

11. Suppose that \(\ln 2 = a\) and \(\ln 5 = b\). Use properties of logarithms to write the given logarithm in terms of \(a\) and \(b\).
    a) \(\ln 20\)
    b) \(\ln 2.5\)
    c) \(\ln \sqrt{5}\)

12. Solve the equation. Give exact values, do not use a calculator.
    \(2^{1-x} = \left(\frac{3}{5}\right)^x\)

13. Change the logarithmic expression to an equivalent exponential expression
    a) \(\ln \left(\frac{1}{e^5}\right) = -5\)
    b) \(\log_b 8 = 3\)
14. Graph the given function using transformations. Plot at least 3 points on the graph of the basic function and use them to perform transformations. Do one transformation at a time and write the equation for each function.
   a) \( f(x) = \log_2(-x + 2) \)

   \[
   \begin{array}{c}
   \text{Graph of } f(x) = \log_2(-x + 2) \\
   \end{array}
   \]

   b) \( f(x) = 2 - \ln(x + 4) \)

   \[
   \begin{array}{c}
   \text{Graph of } f(x) = 2 - \ln(x + 4) \\
   \end{array}
   \]

   c) \( f(x) = e^{\frac{1}{3}x} - 1 \)

   \[
   \begin{array}{c}
   \text{Graph of } f(x) = e^{\frac{1}{3}x} - 1 \\
   \end{array}
   \]

15. Write as the sum and/or difference of logarithms. Express powers as factors. Assume \( x > 9 \).
   a) \( \ln \left( \frac{7x^{\frac{2}{3}}(1 + 6x)}{(x - 9)^2} \right) \)
b) \( \log_3 \left( \frac{(3x-1)^2}{x^2 \sqrt{x-9}} \right) \)

16. Change the exponential expression to equivalent logarithmic expression  \( 5^A = 4 \)

17. Solve logarithmic equations. Give the exact answer. Do not use a calculator. (Don’t forget to consider the domain!)
   a) \( \log_3 x^2 = \log_3 (6x + 7) \)
   b) \( \ln(x - 6) + \ln(x + 1) = \ln(x - 15) \)

18. Let \( f(x) = \log_2 (x - 2) \) and \( g(x) = \log_2 (4x + 16) \). Solve the equation \( f(x) + g(x) = 6 \).

19. Solve the equation. Give exact values. Do not use a calculator
   \( \log_6 (x^2 - x) = 1 \)

20. Let \( f(x) = 2^x + 2^{-x} \) and \( g(x) = 2^x - 2^{-x} \). Find \( [f(x)]^2 - [g(x)]^2 \).

**Answers:**

1) Domain = \((-\infty, \infty)\); range = \((2, \infty)\); asymptote: \( y = 2 \)

2) \( f(x) = e^{x+3} \)

3) Domain = \((-\infty, \infty)\); range = \((-\infty, -3)\); asymptote: \( y = -3 \)
4) a) \( \log_b \frac{t^2 \cdot \sqrt{v}}{\sqrt{u^4} \cdot z^4} \); b) \( \ln \frac{x^2}{y^3} \); c) \( \log(s) \)

5) 1.209

6) a) 1; b) 6; c) 2

7) a) -3, 3; b) 4/3; c) \( \frac{1 + \ln 2}{2} \); d) 7; e) 6; f) 1/8; g) -3, -2; h) \( \frac{\log_2 (3) - 1}{2} \)

8) Shift to the right by 1; horizontal compression by a factor of 2; vertical stretch by a factor of 3; shift up by 4 (other orders are possible)

9) a) -3, b) -1/2

10) a) (-2, 3); b) \((-\infty, -\sqrt{7}) \cup (0, \sqrt{7}), c) (-\infty, -2) \cup (1, +\infty)\)

11) a) 2a+b; b) b-a; c) b/3

12) \( \frac{\ln 2}{\ln(6/5)} \)

13) a) \( \frac{1}{e^5} = e^{-5} \); b) \( b^3 = 8 \)

14) a) 

15) a) \( \ln 7 + \ln x + (1/9) \ln(1+6x) - 7 \ln(x-9) \); b) \( 2 \log_3 (3x - 1) - 2 \log_3 x - \frac{1}{2} \log_3 (x - 9) \)

16) \( A = \log_2 (4) \)

17) a) -1, 7; b) no solution

18) 4

19) -2, 3

20) 4