Review for test 5

1) Write first five terms of the arithmetic sequence for which \( a_1 = 2 \) and \( d = -\frac{4}{3} \)

2) Determine whether given sequence is arithmetic, geometric or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find common ratio
   a) \( a_n = 5^n \)
   b) \( a_n = \left(\frac{4}{3}\right)^n \)

3) Use mathematical induction to prove that the statement is true for every natural number \( n \)
   a) \( 2 \) is a factor of \( n^2 - n + 2 \)
   b) \( 3 + 8 + 13 + \cdots + (5n-2) = \frac{n(5n+1)}{2} \)

4) Sequence \( \{a_n\} \) is arithmetic. Find \( a_{17} \), if \( a_1 = -7 \) and \( d = -2 \).

5) Write the first four terms of the sequence whose general term is given
   a) \( a_n = (-1)^{n+1} (n + 9) \)
   b) \( a_n = \frac{n^2}{(n+1)!} \)
   c) \( a_n = \frac{n + 1}{2n-1} \)

6) Express the given sum using sigma notation
   \[
   \frac{1}{y} + \frac{r}{2y} + \frac{r^2}{3y} + \cdots + \frac{r^{n-1}}{ny}
   \]

7) Find the first term, the common difference and give the recursive formula for the arithmetic sequence whose 10\(^{th}\) term is \(-21\) and 16\(^{th}\) term is \(-39\)

8) Use the Binomial Theorem to expand the binomial and express the result in simplified form
   a) \( (x+2y)^6 \)
   b) \( (x^2 - 5y)^4 \)

9) Determine whether the infinite geometric series converges or diverges. Explain. If it converges, find its sum
   \[
   5 + \frac{5}{4} + \frac{5}{16} + \frac{5}{64} + \cdots
   \]

10) Use the formula for the sum of the first \( n \) terms of a geometric sequence to find the sum of the
    a) first five terms of the geometric sequence \( \frac{3}{2}, \frac{3}{8}, \frac{3}{32}, \cdots \)
    b) First eight terms of the geometric sequence -5, -10, -20, -40, ...

11) Write the first five terms of the geometric sequence for which
    a) \( a_1 = 4 \) and \( r = -2 \)
b) \( a_1 = -3; \ a_n = -3a_{n-1} \)

12) Write the formula for the general term of the arithmetic sequence: -23, -27, -31, -35, ... and use the formula to find the 20th term of the sequence.

13) Write as a sum and find the value of \( \sum_{i=3}^{6}(3i - 5) \)

14) Find the sum of the first 60 terms of the arithmetic sequence: 16, 12, 8, 4, ....

15) Find the common difference for the arithmetic sequence: -15, -17, -19, -21, ....

16) A statement \( P_n \) about the positive integers is given. Write statements \( P_1, P_2, P_3, \) and show that each of these statements is true
   a) \( P_n \): 2 is a factor of \( n^2 + 1\)\( n \)
   b) \( P_n \): \( 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3} \)

17) Find the sum of the infinite geometric series, if it exists
   a) \( 32 + 8 + 2 + \frac{1}{2} + \cdots \)
   b) \( \sum_{i=1}^{n} 10 \left( -\frac{1}{2} \right)^{i-1} \)

18) Find the following sum: \( \sum_{k=1}^{16} (2k + 7) \)

19) Find the common ratio of the geometric sequence
   \( \frac{1}{2}, \frac{3}{4}, \frac{9}{8}, \frac{27}{16}, \cdots \)

20) Write out as the sum. Do not evaluate
   \( \sum_{k=0}^{n-1} (3k + 1) \)

21) Find the 9th term of a geometric sequence for which \( a_1 = 5 \) and \( r = -3 \)

22) Express the sum using sigma notation. Use 1 as the lower limit of summation and \( i \) for the index of summation
   a) \( 3^2 + 6^3 + 9^i + \cdots + 24^9 \)
   b) \( a + 1 + \frac{a + 2}{2} + \frac{a + 3}{3} + \cdots + \frac{a + 6}{6} \)

23) Bob decides to train for a marathon. He begins by jogging 13 minutes one day per week and decides to increase the time by 6 minutes each week. Write the general formula for the number of minutes he will jog in \( n \)-th week and find how many weeks it will take him to run for one hour.

24) Jane put $25 into her bank account on January 1, 2014, $35 on February 1, $45 on March 1, and so forth. If she continues this pattern, how much will she have in her account on Dec 30, 2016?
25) Write a formula for the general term of the geometric sequence: 3, -9, 27, -81,…

26) Find the sum \( \sum_{k=5}^{33} (3k - 1) \)

27) Evaluate the binomial coefficient without using a calculator \( \binom{10}{5} \)

28) Write the first four terms of the sequence given recursively 
   \( a_1 = 2, \ a_2 = 5, \ a_n = a_{n-2} - 3a_{n-1} \)

29) Use the Binomial Theorem to find the coefficient of \( x^6 \) in the expansion of \( (x^2 - 3)^7 \)

30) Use the Binomial Theorem to find the term that contains \( x^9 \) in the expansion of \( (x+3y)^{11} \)

31) Find the coefficient of \( x^5 \) in the expansion of \( (2x - 3)^9 \)

32) Find the sum \( \sum_{k=1}^{13} \left( \frac{2^k}{3} \right) \). Use the formula for the sum of the first \( n \) terms of a geometric sequence.

**Answers:**

1) \( 2, \frac{2}{3}, -\frac{2}{3}, -2, -\frac{10}{3} \)
2) a) neither; b) geometric, \( r = 4/3 \)
3) -39
4) a) 10, -11, 12, -13; b) \( \frac{1}{2}, \frac{2}{3}, \frac{3}{8}, \frac{2}{15} \); c) 2, 1, \( \frac{4}{5}, \frac{5}{7} \)
5) \( \sum_{k=1}^{n} \frac{r^{k-1}}{ky} \)
6) \( a_1 = 6, d = -3, a_{n+1} = a_n - 3 \)
7) \( x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6 \); b) \( x^8 - 20x^6y + 150x^4y^2 - 500x^2y^3 + 625y^4 \)
8) a) converges; \( S = 4 \)
9) 1023/512
10) a) 4, -8, 16, -32, 64; b) -3, 9, -27, 81, -243
11) a) \( a_n = -4n - 19 \); a_{20} = -99
13) $4+7+10+25 = 46$
14) $-6,120$
15) $d = -2$
16) a) $P_1$: 2 is a factor of $1^2 + 11 \cdot 1$; true
   $P_2$: 2 is a factor of $2^2 + 11 \cdot 2$; true
   $P_3$: 3 is a factor of $3^2 + 11 \cdot 3$; true
b) $P_1$: $1 \cdot 2 = \frac{1(1 + 1)(1 + 2)}{3}$, true
   $P_2$: $1 \cdot 2 + 3 = \frac{2(2 + 1)(2 + 2)}{3}$; true
   $P_3$: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 = \frac{3(3 + 1)(3 + 2)}{3}$; true
17) a) $\frac{128}{3}$; b) $\frac{20}{3}$
18) 384
19) $r = -\frac{3}{2}$
20) $1+4+7+ \ldots +(3n-2)$
21) 32,805
22) a) $\sum_{i=1}^{8} (3i)^{i+1}$; b) $\sum_{i=1}^{6} \frac{a + i}{i}$
23) $6n+7$; 9 weeks
24) 3,360
25) $a_n = (-1)^{n+1} \cdot 3^n$
26) 1,624
27) 252
28) 2, 5, -13, 44
29) -945
30) $495x^2y^2$
31) 326,592
32) $\frac{254}{3}$