Review for test 4
1) Write the formula for the general term of the arithmetic sequence: -23, -27, -31, -35, …
   and use the formula to find the 20th term of the sequence.

2) Find the sum \( \sum_{k=5}^{33} (3k - 1) \)

3) Find the center, vertices, foci, transverse axis and the equations of asymptotes of given hyperbolas
   a) \( x^2 - 4y^2 - 2x - 24y - 39 = 0 \)
   b) \( 9y^2 - 16 x^2 = 144 \)
   c) \( \frac{(x+2)^2}{4} - \frac{(y-3)^2}{9} = 1 \)

4) Find the equation, in the standard form, of the hyperbola with vertices (0,± 4) and asymptotes \( y = \pm \frac{1}{2}x \)

5) Write first five terms of the arithmetic sequence for which \( a_1 = 2 \) and \( d = -\frac{4}{3} \)

6) Find the equation, in the standard form, of the hyperbola with vertices (± 5,0) and foci (±8, 0)

7) Find the equation, in the standard form, of the hyperbola with center (3,2), vertex at (3,4) and focus at (3, -3)

8) Write the first four terms of the sequence whose general term is given
   a) \( a_n = (-1)^{n+1} (n + 9) \)
   b) \( a_n = \frac{n^2}{(n+1)!} \)
   c) \( a_n = \frac{n+1}{2n-1} \)

9) Find the asymptotes of the following hyperbolas
   a) \( \frac{x^2}{16} - \frac{y^2}{4} = 1 \)
   b) \( \frac{(y-1)^2}{4} - \frac{(x+3)^2}{25} = 1 \)

10) Express the given sum using sigma notation
   \( \frac{1}{y} + \frac{r}{2y} + \frac{r^2}{3y} + \cdots + \frac{r^{n-1}}{ny} \)

11) Write out as a sum and find the exact value
   a) \( \sum_{i=3}^{6} (3i - 5) \)
   b) \( \sum_{i=0}^{4} \frac{(i + 2)!}{i!} \)
12) Find the vertex, focus and the directrix of the parabola $y^2 = -12x$. Graph the parabola.

13) Find the standard equation of the parabola with vertex at (5,3) and focus at (2,3). Graph the parabola.

14) Find the first term, the common difference and give the recursive formula for the arithmetic sequence whose 10th term is -21 and 16th term is -39.

15) Find the vertex, focus and the directrix of the parabola given by $y^2 - 8y - 8x = -40$. Graph the equation.
16) Graph the equation \( \frac{y^2}{9} - \frac{x^2}{4} = 1 \)

17) Write out as the sum. Do not evaluate
\[
\sum_{k=0}^{n-1} (3k + 1)
\]

18) Graph the equation \( \frac{(x-1)^2}{9} - \frac{(y+1)^2}{25} = 1 \)

19) Express the sum using sigma notation. Use 1 as the lower limit of summation and i for the index of summation
a) \( 3^2 + 6^3 + 9^4 + \ldots + 24^9 \)

b) \( a + 1 + \frac{a + 2}{2} + \frac{a + 3}{3} + \ldots + \frac{a + 6}{6} \)

20) Graph the equation \( 36x^2 = 4y^2 + 144 \)

21) Find the sum of the first 60 terms of the arithmetic sequence: 16, 12, 8, 4, ….
22) Bob decides to train for a marathon. He begins by jogging 13 minutes one day per week and decides to increase the time by 6 minutes each week. Write the general formula for the number of minutes he will jog in n-th week and find how many weeks it will take him to run for one hour.

23) Find the vertex, focus and the directrix of the parabola \((x-2)^2 = -8(y+3)\). Graph the parabola.

\[ \begin{align*}
  \text{Vertex: } & (2, -3) \\
  \text{Focus: } & (2, -3 - \frac{1}{2}) \\
  \text{Directrix: } & y = -3 + \frac{1}{2}
\end{align*} \]

![Graph of the parabola](image)

24) Find the common difference for the arithmetic sequence: -15, -17, -19, -21, ....

25) Find the standard equation of the parabola with vertex at (0,0), x-axis as the axis of symmetry and passing through the point (4,9)

26) Write the first four terms of the sequence given recursively
\( a_1 = 2, \ a_2 = 5, \ a_n = a_{n-2} - 3a_{n-1} \)

27) Write the equation, in the standard form, of the hyperbola whose graph is given below.

![Graph of the hyperbola](image)

28) Jane put $25 into her bank account on January 1,2014, $35 on February 1, $45 on March 1, and so forth. If she continues this pattern, how much will she have in her account on Dec 30, 2016?
29) Graph the equation $x^2 = 6y$

![Graph of the equation $x^2 = 6y$](image)

30) Find the standard equation of the parabola with vertex at (6,-2) and focus at (6,-9).

31) Sequence $\{a_n\}$ is arithmetic. Find $a_1$, if $a_1 = -7$ and $d = -2$.

32) Find the equation of the hyperbola with center at (0,0), focus at $(2\sqrt{5},0)$ and vertex at (4,0). Graph it.

33) Find the following sum: $\sum_{k=1}^{16} (2k + 7)$

34) Write the first four terms of the sequence $\left\{ \frac{3n}{(n+1)!} \right\}_{n=1}^{\infty}$

35) Find the standard equation of the parabola with focus at (-4,0) and the directrix the line $x = 4$.

36) Find the standard equation of the parabola with the focus at (0,3) and the directrix, the line $y = -3$.

Give the two points that define the latus rectum.

37) Use the definition to check whether the given sequences are arithmetic
   a) $\{3n - 9\}_{n=1}^{\infty}$
   b) $\{n^2 - 1\}_{n=1}^{\infty}$

38) The graph of the sequences $\{a_n\}$ and $\{b_n\}$ are given below. Find the indicated sums

a) $\sum_{k=1}^{5} (a_k + 2b_k)$

b) $\sum_{k=1}^{5} \left( \frac{a_k}{b_k^2} \right)$

![Graph of sequences $\{a_n\}$ and $\{b_n\}$](image)
Answers:

1) \(a_n = -4n - 19\); \(a_{20} = -99\)

2) 1,624

3) a) center (1, -3); vertices: (-1, -3), (3, -3); foci: \([1 - \sqrt{5}, -3], [1 + \sqrt{5}, -3]\); transverse axis: \(y = -3\); asymptotes: 
   \[y = \frac{1}{2}x - \frac{5}{2}, y = \frac{1}{2}x - \frac{7}{2}\]
   b) center (0, 0); vertices: (0, -4), (0, 4); foci: (0, -5), (0, 5); transverse axis: \(y = 0\); asymptotes: \(y = \pm \frac{4}{3}x\)
   c) center (-2, 3); vertices: (-4, 3), (0, 3); foci: \([-2 - \sqrt{3}, 3], [-2 + \sqrt{3}, 3]\); transverse axis: \(y = 3\); asymptotes: 
   \[y = -\frac{3}{2}x, y = \frac{3}{2}x + 6\]

4) \(\frac{y^2}{16} - \frac{x^2}{64} = 1\)

5) \(2, \frac{2}{3}, -\frac{2}{3}, -2, -\frac{10}{3}\)

6) \(\frac{x^2}{25} - \frac{y^2}{39} = 1\)

7) \(\frac{(y - 2)^2}{4} - \frac{(x - 3)^2}{21} = 1\)

8) a) 10, -11, 12, -13; b) \(\frac{1}{2}, \frac{2}{3}, \frac{3}{8}, \frac{2}{15}\); c) \(2, \frac{4}{5}, \frac{5}{7}\)

9) a) \(y = \pm \frac{1}{2}x\); b) \(y = -\frac{2}{5}x - \frac{3}{5}, y = \frac{2}{5}x + \frac{11}{5}\)

10) \(\sum_{k=1}^{n} \frac{r^{k-1}}{k}\)

11) a) 4 + 7 + 10 + 25 = 46; b) \(\frac{2!}{0!} + \frac{3!}{1!} + \frac{4!}{2!} + \frac{5!}{3!} + \frac{6!}{4!} = 70\)

12) Vertex: (0, 0); focus: (-3, 0); directrix: \(x = 3\)

13) \((y - 3)^2 = -12 (x - 5)\)

14) \(a_1 = 6, d = -3, a_{n+1} = a_n - 3\)
15) Vertex: (3,4); focus: (5,4); directrix: x = 1

16)

17) $1 + 4 + 7 + \ldots + (3n-2)$

18)

19) a) $\sum_{i=1}^{8} (3i)^{i+1}$; b) $\sum_{i=1}^{6} \frac{a + i}{i}$

20)

21) $-6,120$

22) $6n+7; 9$ weeks
23) Vertex: (2, -3); focus: (2, -5); directrix: \( y = -1 \)

\[
\begin{align*}
\text{Vertex: } (2, -3) &; \text{ focus: } (2, -5) & \text{directrix: } y &= -1 \\
\end{align*}
\]

24) \( d = -2 \)

25) \( y^2 = \frac{81}{4} x \)

26) 2, 5, -13, 44

27) \[
\frac{x^2}{9} - \frac{y^2}{16} = 1
\]

28) 7,200

29) \[
\begin{align*}
\end{align*}
\]

30) \[(x - 6)^2 = -28 (y + 2)\]

31) -39

32) \[
\frac{x^2}{16} - \frac{y^2}{4} = 1
\]

33) 384

34) \[
\frac{3}{2}, \frac{3}{8}, \frac{1}{10}
\]

35) \( y^2 = -16x \)

36) \( x^2 = 12y; (6,3),(-6,3) \)

37) a) yes, b) no

38) a) -10; b) 65/18