

(55 pts.) MWF

MAC 1114

EXAM III

MR. NADEL

SPRING 2016

(10) ① a) If $\cos x = -\frac{3}{7}$,
 $\frac{\pi}{2} \leq x \leq \pi$, Find the exact
value of $\cos 2x$.

b) Find the exact value of
 $\cos 105^\circ$ using a half angle
formula.

(10) ② Prove

$$\tan x \sin 2x = 2 - 2 \cos^2 x$$

(10) ③ Use the formulas provided
on format sheet:

a) Find $\cos 195^\circ \cdot \cos 105^\circ$
exactly.

b) Write as a product
and simplify answer:
 $\sin 7x - \sin 3x$.

(25) ④ a) Solve

4 pts. $\sin \theta = -0.315$
on $0^\circ \leq \theta \leq 360^\circ$.

b) Solve $2 \sin 2x = \sqrt{2}$,
5 pts. on $0 \leq x \leq \pi$, exactly.

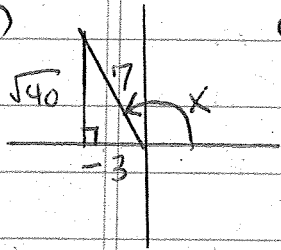
c) Solve $2 \cos^2 x - \sin x = 1$
8 pts. on $0^\circ \leq x \leq 360^\circ$.

d) Solve $\sin 2x = \sin x$,
8 pts. on $-\pi \leq x \leq \pi$,
exactly.

MWF

MAC 1114 EXAM III KEY (SP'16)

① a)



$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(-\frac{3}{7}\right)^2 - \left(\frac{\sqrt{40}}{7}\right)^2 \\ &= \frac{9-40}{49} = -\frac{31}{49}\end{aligned}$$

b)

$$\begin{aligned}& -\sqrt{\frac{1+\cos 210^\circ}{2}} \\ &= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} \\ &= -\sqrt{\frac{2-\sqrt{3}}{2}}\end{aligned}$$

$$\textcircled{2} \tan x \sin 2x = \frac{\sin x}{\cos x} \cdot 2 \sin x \cos x$$

$$\begin{aligned}&= 2 \sin^2 x \\ &= 2(1-\cos^2 x) = 2-2\cos^2 x\end{aligned}$$

$$\begin{aligned}\textcircled{3} \text{ a) } & \frac{1}{2} [\cos(195^\circ+105^\circ) + \cos(195^\circ-105^\circ)] \\ &= \frac{1}{2} [\cos 300^\circ + \cos 90^\circ] \\ &= \frac{1}{2} \left[\frac{1}{2} + 0\right] = \frac{1}{4}\end{aligned}$$

$$\begin{aligned}\text{b) } & 2 \cos\left(\frac{7x+3x}{2}\right) \sin\left(\frac{7x-3x}{2}\right) \\ &= 2 \cos 5x \sin 2x\end{aligned}$$

$$\begin{aligned}\textcircled{4} \text{ a) } & \alpha = \sin^{-1}(+.315) \\ &= 18.4^\circ\end{aligned}$$

$$\theta = 180^\circ + 18.4^\circ = 198.4^\circ \text{ (QIII)}$$

$$\theta = 360^\circ - 18.4^\circ = 341.6^\circ \text{ (QIV)}$$

$$\textcircled{4} \text{ b) } \sin 2x = \frac{\sqrt{2}}{2}, 0 \leq 2x \leq 2\pi$$

$$2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{8}$$

$$2x = \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{8}$$

$$\text{c) } 2(1-\sin^2 x) - \sin x = 1$$

$$2 - 2\sin^2 x - \sin x = 1$$

$$0 = 2\sin^2 x + \sin x - 1$$

$$0 = (2\sin x - 1)(\sin x + 1)$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$

$$x = 30^\circ, 150^\circ \quad x = 270^\circ$$

$$\text{d) } 2 \sin x \cos x = \sin x$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$$\sin x = 0 \quad \cos x = \frac{1}{2}$$

$$x = 0, \pm\pi \quad x = \pm\frac{\pi}{3}$$