

(10) (1) a) If $\cos x = \frac{2}{3}$, $\frac{3\pi}{2} < x < 2\pi$,
Find the exact value of $\sin 2x$.

b) If $\cos x = \frac{3}{5}$, $\frac{3\pi}{2} < x < 2\pi$,
Find the exact value of $\cos\left(\frac{x}{2}\right)$.

(10) (2) Prove $\cos 2x + 2\sin^2 x = 2$.

(10) (3) Use the formulas provided on format sheet:

a) $\cos 105^\circ - \cos 15^\circ$ exactly.

b) Write $\cos 4x \cos 3x$ as a sum. Simplify answer.

(25) (1) a) Solve $\sin x = -.378$
4 pts. on $0 \leq x \leq 2\pi$.

b) Solve $\tan 2\theta = \sqrt{3}$,
5 pts. $0^\circ \leq \theta \leq 360^\circ$.

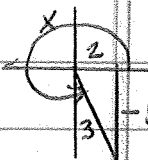
c) Solve
8 pts. $2\sin^2 x - 5\sin x + 2 = 0$
on $0 \leq x \leq 2\pi$.

Hint: factor.

d) Solve $\cos 2\theta = \cos \theta$
8 pts. on $0^\circ \leq \theta \leq 360^\circ$.

Hint: Use a double angle formula for d)

MAC 1114 EXAM III KEY (SP '17)

① a)  $\sin 2x = 2 \sin x \cos x$
 $= 2 \left(-\frac{\sqrt{5}}{3} \right) \left(\frac{2}{3} \right)$
 $= -\frac{4\sqrt{5}}{9}$

b) $\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1+\cos x}{2}}$
 $\frac{3\pi}{4} < \frac{x}{2} < \pi \Rightarrow \frac{x}{2}$ is in QII

$$\cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\frac{2}{3}}{2}}$$

$$= -\sqrt{\frac{5+3}{10}} = -\frac{2}{\sqrt{5}}$$

② $1 + \cos 2x + 2 \sin^2 x =$
 $1 + \cos^2 x - \sin^2 x + 2 \sin^2 x =$
 $1 + (\cos^2 x + \sin^2 x) = 1 + 1 = 2$

③ a) $-2 \sin\left(\frac{105^\circ + 15^\circ}{2}\right) \sin\left(\frac{105^\circ - 15^\circ}{2}\right)$
 $= -2 \sin 60^\circ \sin 45^\circ$
 $= -2 \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{6}}{2}$

b) $\frac{1}{2} [\cos(4x+3x) + \cos(4x-3x)]$
 $= \frac{1}{2} [\cos 7x + \cos x]$

This doesn't simplify more!

④ a) $\alpha = \text{ref. angle}$
 $= \sin^{-1}(+0.378) \approx 0.3876$

$\pi + \alpha \approx 3.5292$ (QIII)

$2\pi - \alpha \approx 5.8956$ (QIV)

④ b) search interval:

$$0^\circ \leq \theta \leq 720^\circ$$

$$2\theta = 60^\circ \Rightarrow \theta = 30^\circ$$

$$2\theta = 240^\circ \Rightarrow \theta = 120^\circ$$

$$2\theta = 420^\circ \Rightarrow \theta = 210^\circ$$

$$2\theta = 600^\circ \Rightarrow \theta = 300^\circ$$

④ c) $(2 \sin x - 1)(\sin x - 2) = 0$

$$\sin x = \frac{1}{2}$$

$$\sin x = 2$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

impossible

④ d) $\cos^2 \theta - \sin^2 \theta = \cos \theta$

$$\cos^2 \theta - (1 - \cos^2 \theta) = \cos \theta$$

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$2 \cos \theta + 1 = 0$$

$$\cos \theta - 1 = 0$$

$$\cos \theta = -\frac{1}{2}$$

$$\cos \theta = 1$$

$$\theta = 120^\circ, 240^\circ$$

$$\theta = 0^\circ, 360^\circ$$