

Some Techniques of Integration

① Integration by Parts

$$\int u dv = uv - \int v du$$

For integrals like $\int x e^x dx$, $\int \ln x dx$, $\int \tan^{-1} x dx$.

But use u-substitution on $\int x e^{(x^2)} dx$.

② Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad \cot^2 x + 1 = \csc^2 x.$$

③ Half Angle Formulas

Used for even powers of sine and cosine, with no odd powers appearing.

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

④ "odd man out" for odd power on sine or cosine.
TOLES Don't need reduction formula for odd power on tangent or even power on secant.

⑤ Trigonometric Substitutions

$$\sqrt{a^2 - x^2} \quad \text{Let } x = a \sin \theta$$

$$\sqrt{a^2 + x^2} \quad \text{Let } x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \quad \text{Let } x = a \sec \theta$$

Sometimes we need $\sin 2\theta = 2 \sin \theta \cos \theta$.

⑥ Sample Partial Fractions Need proper rational function.

$$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3}, \quad \frac{Ax+B}{x^2+1}, \quad \frac{Ax+B}{x^2+x+1}$$

of constants = degree of denominator
irreducible quadratic ax^2+bx+c if $b^2-4ac < 0$.

⑦ Completing the square: $|x^2 + ax + \left(\frac{a}{2}\right)^2$. Need a 1 in front of x^2 . Otherwise factor out of first 2 terms.

⑧ Difference between inverse tangents and logarithms:

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C, \quad \int \frac{x}{x^2+a^2} dx = \frac{1}{2} \ln(x^2+a^2) + C$$