Ex. 5 (logarithmic)

Sketch $y = f(x) = \frac{\ln x}{x}$

Defined if $x > 0$.

No symmetry. $\frac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \Rightarrow x = e^0 = 1$ (x intercept)

No y intercept.

$\lim_{x \to 0^+} \frac{\ln x}{x} = -\infty$ since $\lim_{x \to 0^+} \ln x = -\infty$ (not indeterminate)

There is a VA at $x = 0$

$\lim_{x \to \infty} \frac{\ln x}{x} = 0$ by L.R. $\left(\lim_{x \to \infty} \frac{\left(\frac{1}{x}\right)}{1} = 0\right)$

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \ln x (1)}{x^2} = \frac{1 - \ln x}{x^2}$$

$1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

0 inc. e dec. (+ to the left of e, − to the right)

$f(e) = \frac{\ln e}{e} \approx 0.37$

$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \ln x)(2x)}{x^4} = \frac{2x \ln x - 3x}{x^4}$$

$$= \frac{2 \ln x - 3}{x^3}$$

Since $x > 0$, we need only look at $2 \ln x - 3$

0 c.d. $e^{3/2}$ c.u.

$f(e^{3/2}) = \frac{3}{2e^{3/2}} \approx 0.33$ ($e^{3/2}$ approx. 4.48)