FACTORING A QUADRATIC FORM

To factor a quadratic form $ax^2 + bx + c$, $a \neq 1$, $b, c \neq 0$ you may follow this procedure:

1. Find the product *ac* .
2. Look for 2 factors of *ac* that combine [add or subtract] to give *b* [and their product is *ac - with the correct sign.*]
3. Rewrite the quadratic as a 4-term polynomial by breaking the middle term up into 2 terms, expressing *bx* as the sum or difference of *x* times each of the 2 factors you found in step 2. [It makes no difference in which order you write these 2 terms.]
4. Group the first 2 terms together [mentally] and group the last two terms together [mentally].
5. Factor out the GCF of each of those two groups of terms.
6. The binomial remaining in parentheses will be the same for each term. Factor out this common binomial, leaving you with a second binomial.
   [Recall the following distributive property; $A(B+C)+D(B+C)=(A+D)(B+C)$]

EXAMPLE 1: $5x^2 - 13x + 6$

1. $ac = 5(6) = 30$
2. The 2 factors of 30 that combine to give -13 are: -10 and -3 [with product 30]
   [Note: if you try to use -15 and 2, they combine to give -13, but their product is -30 instead of 30]
3. So $5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$
4. $= (5x^2 - 10x) + (-3x + 6)$
5. $= 5x(x-2) - 3(x-2)$ [Note: we factored out -3 instead of 3 in the second term from step 4 to give us the factor $(x-2)$ instead of $(2-x)$]
6. $= (5x-3)(x-2)$

EXAMPLE 2: $3x^2 - 2x - 5$

1. $ac = 3(-5) = -15$
2. 2 factors of -15 that combine to give -2 are: -5 and 3.
3. So $3x^2 - 2x - 5 = 3x^2 - 5x + 3x - 5$
4. $= x(3x-5) + 3x - 5 = x(3x-5) + 1(3x-5)$ [Note: there wasn't anything but +1 to factor out of the second term.]
5. $= (x+1)(3x-5)$

NOTE: If there are <u>no</u> factors of *ac* that combine to equal *b* then the quadratic form is <u>prime</u> [has no factors] relative to the integers.

e.g. Consider $3x^2 + 3x + 2$. $ac = 6$, but there are not 2 factors of 6 that combine to equal 3 ; hence this quadratic form is prime relative to the integers.