Chap. 12 Practice Problems

① a) Convert $(2, \frac{2\pi}{3})$ to rectangular coordinates.
b) Determine all primary representations in polar coordinates of $(\sqrt{3}, 1)$.

② Show that $x^2 - y^2 = 1$ has polar equation
$$r^2 = \sec 2\theta$$

③ Sketch $r = \sin 2\theta$

④ Sketch $r = 1 + 2\sin\theta$

⑤ Find the area of the region bounded by one petal of $r = \cos 2\theta$.

⑥ Find the area bounded by
$$r = 2(1 + \sin\theta)$$

Chap. 12 Solutions

① a) $x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

$y = 2\sin\frac{2\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

b) $r^2 = (\sqrt{3})^2 + 1^2 = 4 \qquad r = \pm 2$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\left(2, \frac{\pi}{6}\right)$ OR $\left(-2, \frac{7\pi}{6}\right)$

② $x^2 - y^2 = 1$

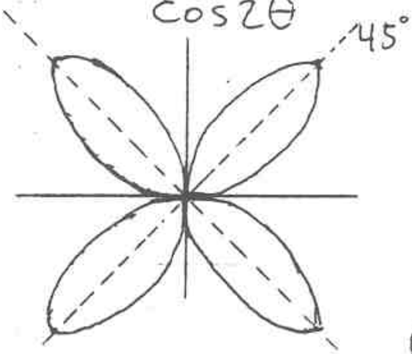
$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

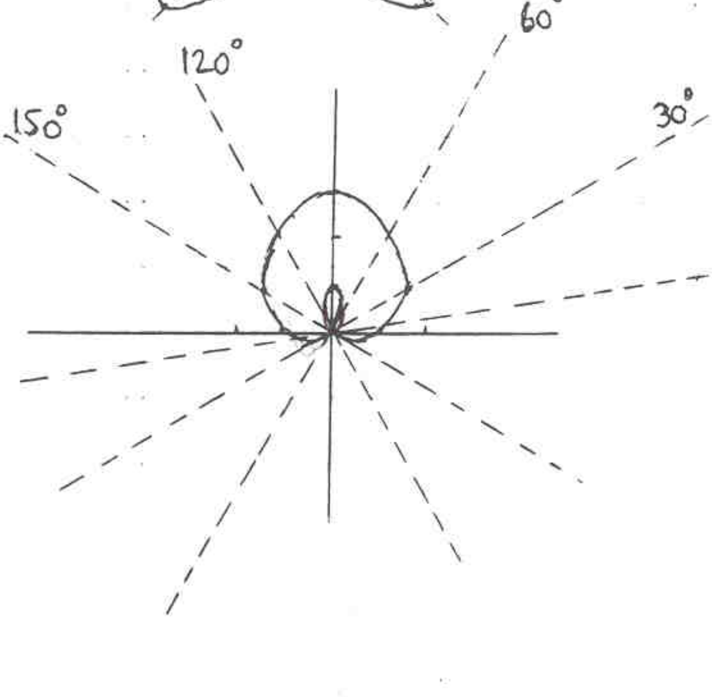
$r^2\cos 2\theta = 1$

$r^2 = \frac{1}{\cos 2\theta} = \sec 2\theta$

③



④



⑤ $2 \cdot \frac{1}{2}\int_0^{\pi/4} (\cos 2\theta)^2\,d\theta$ ← symmetry

$= \int_0^{\pi/4} \cos^2 2\theta\,d\theta$

$= \int_0^{\pi/4} \frac{1+\cos 4\theta}{2}\,d\theta$

$= \frac{1}{2}\left[\theta + \frac{\sin 4\theta}{4}\right]\Big|_0^{\pi/4}$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$

⑥ $\frac{1}{2}\int_0^{2\pi}\left[2(1+\sin\theta)\right]^2 d\theta =$

$2\int_0^{2\pi}\left[1 + 2\sin\theta + \sin^2\theta\right]d\theta =$

$2\int_0^{2\pi}\left[1 + 2\sin\theta + \frac{1-\cos 2\theta}{2}\right]d\theta$

$= 2\int_0^{2\pi}\left[\frac{3}{2} + 2\sin\theta - \frac{\cos 2\theta}{2}\right]d\theta$

$= 2\left[\frac{3}{2}\theta - 2\cos\theta - \frac{\sin 2\theta}{4}\right]\Big|_0^{2\pi}$

$= 6\pi$

Introduction to Polar Coordinates

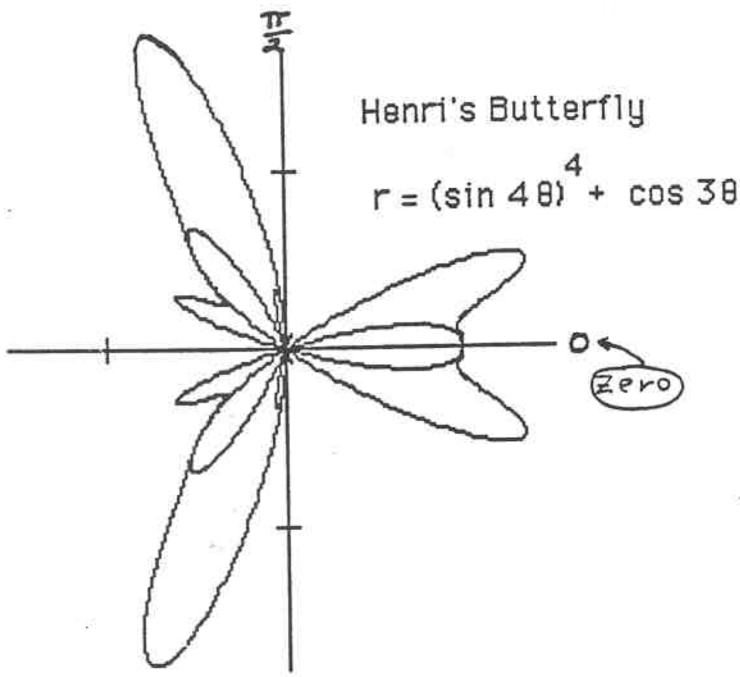


Figure 1

This polar coordinate graph was discovered by Henri Berger, a student in one of the author's (David Cohen) mathematics classes at UCLA, Spring 1988.

Polar Graphing Grid