MAC 2312 EXAM I MR. NADEL SPRING 2013

(5) ① $\int \left(\frac{4x^4+1}{2x} + e^{3x} \right) dx$

(5) ② $\int \cos^3\theta \sin\theta \, d\theta$

(15) ③ a) Evaluate $\sum_{K=1}^{25} (K+2)(2K-1)$ using formulas on the board.

b) Express $1\cdot 2 - 3\cdot 4 + 5\cdot 6 - 7\cdot 8$ in sigma notation.

c) If $f(x) = x^2$ on $[2, 5]$, $n=5$, evaluate

$$\sum_{K=1}^{5} f(m_K)\Delta x$$

where m_K is the midpoint of the K^{th} subinterval.

(15) ④ a) Let $f(x) = x+1$, $a=-5$, $b=1.5$. Suppose $\Delta x_1 = 2$, $\Delta x_2 = 1$, $\Delta x_3 = 3$, $\Delta x_4 = .5$, $x_1^* = -4$, $x_2^* = -2.5$, $x_3^* = 0$, $x_4^* = 1$. Find the values of

$$\sum_{K=1}^{4} f(x_K^*)\Delta x_K \text{ and } \max \Delta x_K$$

b) Find $\int_{-2}^{1} (x+1)\,dx$ using formula(s) from geometry.

c) Express $\int_0^2 (2x-3x^2)\,dx$ as the limit of a sum. You may assume equal subintervals. Do not evaluate.

(10) ⑤ a) $\int_0^{\pi/3} \frac{1}{\cos^2\theta}\,d\theta$

⑤ b) $\int_{-2}^{5} |x-1|\,dx$ algebraically, without geometry

(10) ⑥ a) Find the average value of $f(x) = x^3+1$ on $[-1, 2]$

b) Find all values of x^* guaranteed by the Mean Value Thm. for Integrals for part ⑥ a). (Use calculator.)

(5) ⑦ Find $\frac{d}{dx}\int_1^{x^3} \frac{\sin t}{t}\,dt$

(5) ⑧ $\int_1^e \frac{(\ln x)^2}{x}\,dx$

(5) ⑨ Prove that the sum of the first n positive integers is

$$\frac{n(n+1)}{2}$$

MAC 2312 EXAM I KEY (SP '13)

① $\int\left(2x^3 + \frac{1}{2x} + e^{3x}\right)dx$

$= \frac{x^4}{2} + \frac{1}{2}\ln|x| + \frac{1}{3}e^{3x} + C$

② $u = \cos\theta \quad du = -\sin\theta\, d\theta$

$-du = \sin\theta\, d\theta$

$-\int u^3\, du = -\frac{u^4}{4} + C$

$= -\frac{\cos^4\theta}{4} + C$

③a) $\sum_{k=1}^{25}\left(2k^2 + 3k - 2\right)$

$= 2\sum_{k=1}^{25} k^2 + 3\sum_{k=1}^{25} k - \sum_{k=1}^{25} 2$

$= \frac{2(25)(26)(51)}{6} + \frac{3(25)(26)}{2} - 50$

$= 11,975$

b) $\sum_{k=1}^{4}(-1)^{k+1}(2k-1)(2k)$ OR

$\sum_{k=0}^{3}(-1)^{k}(2k+1)(2k+2)$

c) $\Delta x = \frac{5-2}{5} = .6$

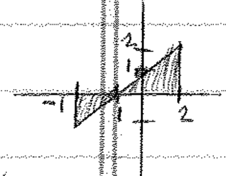
$.6\left[2.3^2 + 2.9^2 + 3.5^2 + 4.1^2 + 4.7^2\right]$

$= 38.91$

④a) $f(-4)\cdot(2) + f(-2.5)\cdot(1) + f(0)\cdot(3) + f(1)(.5)$

$= -6 - 1.5 + 3 + 1 = -3.5$

b)



The integral "sees" the first triangle as "negative"

$-\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = -\frac{1}{2} + 2 = \frac{3}{2}$

c) $\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n}\left(2x_k^* - 3(x_k^*)^2\right)\Delta x_k$

OR $\lim_{\boxed{n\to\infty}} \sum_{k=1}^{n}\left(2x_k^* - 3(x_k^*)^2\right)\Delta x$

or $\Delta x \to 0$

⑤a) $\int_0^{\pi/3} \sec^2\theta\, d\theta$

$= \tan\theta \Big|_0^{\pi/3}$

$= \tan\frac{\pi}{3} - \tan 0 = \sqrt{3}$

b) $\int_{-2}^{1}(1-x)\,dx + \int_1^5 (x-1)\,dx$

$= \left(x - \frac{x^2}{2}\right)\Big|_{-2}^{1} + \left(\frac{x^2}{2} - x\right)\Big|_1^5$

$= \left(\left(1-\frac{1}{2}\right) - (-2-2)\right) + \left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)$

$= \frac{25}{2}$

⑥a) $\frac{1}{3}\int_{-1}^{2}(x^3+1)\,dx = \frac{1}{3}\left[\frac{x^4}{4} + x\right]_{-1}^{2}$

$= \frac{1}{3}\left[4 + 2 - \left(\frac{1}{4} - 1\right)\right] = \frac{9}{4} = 2.25$

b) $x^3 + 1 = \frac{9}{4}$

$x^3 = \frac{5}{4} \Rightarrow x = \sqrt[3]{\frac{5}{4}} \approx 1.077$

⑦ $\frac{\sin(x^3)}{x^3} \cdot 3x^2$

C.R.

⑧ $u = \ln x$

$du = \frac{1}{x}\,dx$

$\int_0^1 u^2\, du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$

⑨ See text or notes.