(75 pts.)

MAC 2312 EXAM I MR. NADEL FALL 2017

(5) ① $\int (x^3-1)^2\,dx$

(5) ② $\int \frac{e^x}{\sqrt{9-e^{2x}}}\,dx$

Hint. $e^{2x} = (e^x)^2$

(15) ③ a) Evaluate

$\sum_{K=1}^{25} (K^2 - 2K + 2)$

using formulas on the board.

b) Express

$-1+3-5+7-9$

in sigma notation.

Do not evaluate the sum.

c) If $f(x) = x^3$ on $[2,5]$, $n=5$, evaluate

$\sum_{K=1}^{5} f(x_{K-1})\Delta x$

(15) ④ a) Let $f(x) = x^3$, $a=-3$, $b=3$. Suppose $\Delta x_1 = 2$, $\Delta x_2 = 1$, $\Delta x_3 = 1$, $\Delta x_4 = 2$, $x_1^* = -2$, $x_2^* = 0$, $x_3^* = 0$, $x_4^* = 2$

Find the values of

$\sum_{K=1}^{4} f(x_K^*)\Delta x_K$ and

$\max \Delta x_K$

④ b) Find $\int_0^3 \sqrt{9-x^2}\,dx$

using formula(s) from geometry.

c) Express

$\lim_{\max \Delta x_K \to 0} \sum_{K=1}^{n} \left[(x_K^*)^5 - 2(x_K^*)\right]\Delta x_K$

$a=0$, $b=1$, as a definite integral

(10) ⑤ a) $\int_0^{\pi/4} (\sec^2\theta + \sin\theta)\,d\theta$

b) $\int_1^9 \left(\frac{1}{\sqrt{t}} - 3\sqrt{t}\right)dt$

(10) ⑥ a) Find the average value of $f(x) = x^2$ on $[-1,2]$.

b) Find all values of x^* guaranteed by the Mean Value Thm for Integrals for part ⑥a)

(5) ⑦ If $F(x) = \int_0^{x^2} \frac{1}{\sqrt{1+3t^2}}\,dt$

find $F'(x)$.

(5) ⑧ $\int_0^1 \frac{x}{(1+x^2)^2}\,dx$

(5) ⑨ Prove that the sum of the first n positive integers is $\frac{n(n+1)}{2}$.

(Do not use induction.)

MAC 2312 EXAM I KEY (F'17)

① $\int (x^6 - 2x^3 + 1)\,dx$

$= \frac{x^7}{7} - \frac{\cancel{2}x^4}{\cancel{4}2} + x + C$

② $u = e^x \quad du = e^x\,dx$

$\int \frac{du}{\sqrt{9-u^2}} = \sin^{-1}\left(\frac{u}{3}\right) + C$

$= \sin^{-1}\left(\frac{e^x}{3}\right) + C$

③ a) $\sum_{k=1}^{25} k^2 - 2\sum_{k=1}^{25} k + \sum_{k=1}^{25} 2$

$= \frac{25(26)(51)}{6} - \frac{2(25)(26)}{2} + 25(2)$

$= 4925$

b) $\sum_{k=1}^{5} (-1)^k(2k-1)$ OR $\sum_{k=0}^{4} (-1)^{k+1}(2k+1)$

c) $\Delta x = \frac{5-2}{5} = .6$

$.6[2^3 + 2.6^3 + 3.2^3 + 3.8^3 + 4.4^3]$

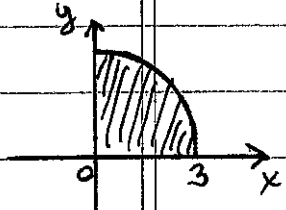
$= 119.04$

④ a) $(-2)^3(2) + 0^3(1) + 0^3(1) + 2^3(2)$

$= 0$

$\max \Delta x_k = 2$

b)



$A = \frac{1}{4}\pi(3^2)$

$= \frac{9\pi}{4}$

≈ 7.069

c) $\int_0^1 (x^5 - 2x)\,dx$

⑤ a) $\int_0^{\pi/4} (\sec^2\theta + \sin\theta)\,d\theta$

$= (\tan\theta - \cos\theta)\Big|_0^{\pi/4}$

$= \tan\frac{\pi}{4} - \cos\frac{\pi}{4} - (\tan 0 - \cos 0)$

$= 1 - \frac{\sqrt{2}}{2} + 1 = 2 - \frac{\sqrt{2}}{2} \approx 1.29$

b) $\int_1^9 (t^{-1/2} - 3t^{1/2})\,dt$

$= \left(2t^{1/2} - \cancel{3}\left(\frac{2}{\cancel{3}}\right)t^{3/2}\right)\Big|_1^9$

$= 2(9)^{1/2} - 2(9)^{3/2} - (2-2) = -48$

⑥ a) $\frac{1}{2-(-1)}\int_{-1}^{2} x^2\,dx$

$= \frac{1}{3}\cdot\frac{x^3}{3}\Big|_{-1}^{2} = \frac{1}{9}(2^3 - (-1)^3) = 1$

b) $x^2 = 1 \Rightarrow x = \pm 1$

⑦ $\frac{1}{\sqrt{1+3(x^2)^2}} \cdot \underset{\text{C.R.}}{2x}$

$= \frac{2x}{\sqrt{1+3x^4}}$

⑧ $\int_0^1 \frac{x}{(1+x^2)^2}\,dx$

$u = 1+x^2, \quad 1\,du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int_1^2 u^{-2}\,du = \frac{1}{2}\cdot\frac{u^{-1}}{-1}\Big|_1^2$

$= -\frac{1}{2}\left(\frac{1}{2} - 1\right) = \frac{1}{4}$

⑨ see notes