(65 pts.)

MAC 2233 EXAM V MR. NADEL SUMMER 2016

(20) ① a) Find the future value of a continuous income stream with a flow rate of \$9,000 per year at 12% compounded continuously for 8 years.

b) The demand and supply functions are

$p = D(q) = 50 - 0.1q$ and

$p = S(q) = 11 + 0.05q$

Find equilibrium price and quantity.

c) Now find the consumers' surplus.

d) Suppose $R'(t) = 5900 - 7t^2$ dollars per year, and $C'(t) = 4118 + 15t^2$ dollars per year where R, C are revenue and cost for a particular machine. (t in years.) Find its useful life.

(15) ② Let $f(x,y) = \dfrac{e^{(x^2+y^2)}}{x-y}$

and $g(x,y) = \ln(x^2 - y)$

a) Find domain of f

b) Find domain of g.

c) Compute $g(3,1)$ on the calculator.

(15) ③ a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = f(x,y) = (x \ln y - y^3)^3$

b) If $z = e^{(x^2 y)}$, $x = t^2$, $y = 2t + 1$, find $\frac{dz}{dt}$ at $t = 1$, using the Chain Rule for partial derivatives

c) If $Q(K, L) = 15K^{0.6}L^{0.4}$ units where K = capital investment in units of \$1,000 and L is the labor force in worker-hours, Find the marginal productivity of capital, Q_K, when the labor force is 4000 and capital investment is \$2,500,000.

(15) ④ A firm produces 2 kinds of calculators, x thousand of type 1 and y thousand of type 2. The profit in millions of dollars is given by $P(x,y) = 12y - 4x - x^2 + 2xy - 2y^2 - 5$. Find the number of each type of calculator should be made to maximize profit. Also find the maximum profit. Justify using the Second Partials Test and show all work.

MAC 2233 EXAM II KEY (SU'16)

① a) $e^{.12(8)}\int_0^8 9000e^{-.12t}\,dt =$

$e^{.96}\cdot \left.\dfrac{9000\,e^{-.12t}}{-.12}\right|_0^8$

$e^{.96}(-75{,}000)(e^{-.96}-1)$

$\$120{,}877.24$

b) $50-0.1q = 11+0.05q$

$39 = .15q$

$q = 260$

$p = 50-0.1(260) = 24$

c) $CS = \int_0^{260}(50-0.1q)\,dq - 24(260)$

$= \left(50q - \dfrac{0.1q^2}{2}\right)\Big|_0^{260} - 6240$

$= \$3{,}380$

d) $5900-7t^2 = 4118+15t^2$

$1782 = 22t^2$

$t^2 = 81 \Rightarrow t = 9$ yrs.

② a) $x-y \neq 0$ OR $x \neq y$

b) $x^2-y>0$ OR $x^2>y$

OR $y<x^2$

c) $g(3,1) = \ln(9-1) = \ln 8$

≈ 2.07944

③ a) $\dfrac{\partial z}{\partial x} = 3(x\ln y - y^3)^2(\ln y)$

C.R.

$\dfrac{\partial z}{\partial y} = 3(x\ln y - y^3)^2\left(\dfrac{x}{y} - 3y^2\right)$

③ b) $t=1 \Rightarrow x=1,\ y=3$

$\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt}$

$= e^{x^2y}(2xy)(2t) + e^{x^2y}(x^2)(2)$

$= e^3\,2(1)(3)(2) + e^3(1)(2)$

≈ 281.2

c) $Q_K = 9K^{-0.4}L^{0.4}$

$= 9\,\dfrac{1}{(2500)^{0.4}}(4000)^{0.4}$

≈ 10.86

④ $P(x,y) = 12y-4x-x^2+2xy-2y^2-5$

$P_x = -4-2x+2y = 0$

$P_y = 12+2x-4y = 0$

$8 \quad -2y = 0$

$y = 4$ thousand

$12+2x = 16$

$2x = 4 \Rightarrow x = 2$ thousand

$P_{max} = 15$ million dollars

$P_{xx} = -2$

$P_{yy} = -4$

$P_{xy} = 2$

$D = (-2)(-4)-2^2$

$= 4>0$

$P_{xx}<0 \Rightarrow$ maximum