MAC 2233 (65 pts.) EXAM IV MR. NADEL SUMMER 2016

(15) ① a) $\int \left(\frac{3}{x} + \frac{3}{x^2} + \frac{3}{e^x}\right) dx$

b) $\int \sqrt{x}\left(8x - \frac{7}{x}\right) dx$

c) The population of a town will be increasing at the rate of $3 + 5x^{3/2}$ people per month, $x$ months from now. Currently the population is 7,000. Find the population in 9 months.

(15) ② a) $\int x^2 \sqrt{x^3+9}\, dx$

b) $\int x\sqrt{x-2}\, dx$ Hint: Let $u = x-2$

c) Water flows into a tank at the rate of $(2t+3)^{1/2}$ ft³/min. If the tank is empty when $t=0$, how much water does it contain 4 min. later? (Answer to 2 dec. places.)

20) ③ a) $\int_1^2 \left(7 - \frac{2}{x}\right) dx$

b) $\int_0^1 \left(x^3 - 3x^2 + e^{-2x}\right) dx$

c) $\int_0^1 \frac{x\, dx}{(2x^2+3)^2}$

(next column)

③ d) It's estimated that $t$ days from now a crop will increase at the rate of

$$0.3t^2 + 0.6t + 1 \quad \frac{\text{bushels}}{\text{day}}.$$

By how much will the crop's value increase during the next 7 days, if the price stays fixed at \$6 per bushel.

(10) ④ Find the <u>area</u> in the <u>first</u> quadrant between $y = x^2$ and $y = x^4$.

(5) ⑤ Find the <u>average value</u> of the function

$$y = f(x) = \frac{3}{x+1}$$

over the interval $1 \le x \le 5$.

# MAC 2233 EXAM IV KEY (SU '16)

① a) $\int\left(\frac{3}{x}+3x^{-2}+3e^{-x}\right)dx$

$=3\ln|x|+\frac{3x^{-1}}{-1}+\frac{3e^{-x}}{-1}+C$

b) $\int x^{1/2}\left(8x-\frac{7}{x}\right)dx$

$=\int\left(8x^{3/2}-7x^{-1/2}\right)dx$

$=8\cdot\frac{2}{5}x^{5/2}-7(2)x^{1/2}+C$

$=\frac{16}{5}x^{5/2}-14x^{1/2}+C$

c) $P=\int\left(3+5x^{3/2}\right)dx$

$=3x+2x^{5/2}+C$

$C=7000$

$P=3x+2x^{5/2}+7000$

When $x=9$, $P=7513$

② a) $u=x^3+9$, $du=3x^2dx$, $\frac{1}{3}du=x^2dx$

$\frac{1}{3}\int u^{1/2}du=\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+C$

$=\frac{2}{9}(x^3+9)^{3/2}+C$

b) $u=x-2\Rightarrow u+2=x$, $du=dx$

$\int(u+2)u^{1/2}du=\int(u^{3/2}+2u^{1/2})du$

$=\frac{2}{5}u^{5/2}+2\cdot\frac{2}{3}u^{3/2}+C$

$=\frac{2}{5}(x-2)^{5/2}+\frac{4}{3}(x-2)^{3/2}+C$

c) $\int(2t+3)^{1/2}dt$

$=\left(\frac{1}{2}\right)\cdot\frac{2}{3}(2t+3)^{3/2}+C$

↑ to balance C.R. $=\frac{1}{3}(2t+3)^{3/2}+C$

$t=0\Rightarrow\frac{1}{3}(3)^{3/2}+C=0$

$\Rightarrow C=-\frac{1}{3}(3)^{3/2}$

$W(t)=\frac{1}{3}(2t+3)^{3/2}-\frac{1}{3}(3)^{3/2}$

$W(4)=\frac{1}{3}(11)^{3/2}-\frac{1}{3}(3)^{3/2}$

$\approx 10.43$

③ a) $(7x-2\ln|x|)\Big|_1^2$

$=14-2\ln 2-(7-2\ln 1)$

$\approx 5.6137$

b) $\left(\frac{x^4}{4}-x^3+\frac{e^{-2x}}{-2}\right)\Big|_0^1$

$=\frac{1}{4}-1+\frac{e^{-2}}{-2}-\left(-\frac{1}{2}\right)\approx -.318$

c) $u=2x^2+3$, $du=4xdx$, $\frac{1}{4}du=xdx$

$\frac{1}{4}\int_3^5 u^{-2}du=-\frac{1}{4}u^{-1}\Big|_3^5$

$=-\frac{1}{20}+\frac{1}{12}=\frac{2}{60}=\frac{1}{30}$

d) $6\int_0^7(0.3t^2+0.6t+1)dt$

$=6(.1t^3+0.3t^2+t)\Big|_0^7$

$=6(.1(7)^3+0.3(7)^2+7)=\$336$

④ $\int_0^1(x^2-x^4)dx=\left(\frac{x^3}{3}-\frac{x^5}{5}\right)\Big|_0^1$

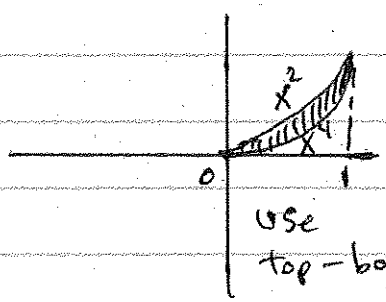
$=\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$

⑤ $\frac{1}{4}\int_1^5\frac{3}{x+1}dx$

$=\frac{3}{4}\ln|x+1|\Big|_1^5$

$=\frac{3}{4}(\ln 6-\ln 2)\approx .824$

Picture for ④

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