(75 pts.) (modified)

MAC 2233 EXAM II MR. NADEL SUMMER 2016

(15) ① a) Suppose the total revenue in dollars, of manufacturing q units and selling them is

$R(q) = 240q + 0.05q^2$.

Use marginal analysis to estimate the additional revenue generated by the sale of the 75^{th} unit.

b) Now compute the actual additional revenue generated by the sale of the 75^{th} unit.

c) A factory produces

$Q(L) = 60{,}000 L^{1/3}$ units, if L denotes the labor force in worker-hours. Estimate the effect on output if L increases from 1,000 to 1,060. (Use calculus.)

(10) ② $q = 300 - 3p$, $0 \le p \le 100$.

a) Express $E(p)$ in terms of p.

b) If $p = 80$, classify demand.

(10) ③ Sketch

$y = f(x) = x^4 - 4x^3 + 2$

Be sure to include intervals of increase, decrease, concavity and y intercept.

Go by 5's on the y axis.

Plot some extra points, too.

(15) ④ Sketch

$$y = f(x) = \frac{x}{x^2 - 4}$$

Hint: $$f'(x) = -\frac{x^2 + 4}{(x^2 - 4)^2}$$

$$f''(x) = \frac{2x(x^2 + 12)}{(x^2 - 4)^3}$$

(10) ⑤ Find absolute maximum and minimum values of

$f(x) = 2x^3 - 7x^2 + 8x + 2$, $0 \le x \le 3$.

(10) ⑥ The total cost in dollars of manufacturing q units of a commodity is

$C(q) = 3q^2 + 3q + 108$

a) At what level of production is the average cost smallest?

b) At what level is

average cost = marginal cost?

(5) ⑦ Let p = unit price of a commodity. If revenue is $R(p) = 300p - p^3$, find the price p which gives the maximum revenue and find the maximum revenue.

Use Calculus

(modified)

MAC 2233 EXAM II KEY (SU'16)

① a) $R'(q) = 240 + .1q$

$R'(74) = 240 + .1(74)$

$= \$247.4$

b) $R(75) - R(74)$

$= 18{,}281.25 - 18{,}033.80$

$= \$247.45$

c) $Q = 60{,}000 L^{1/3}$

$Q' = 20{,}000 L^{-2/3}$

$\Delta Q = Q'(L)\Delta L$

$= \dfrac{20{,}000}{(1000)^{2/3}} \cdot 60$

$= 12{,}000$

② a) $E(p) = -\dfrac{p}{q}\dfrac{dq}{dp}$

$= \dfrac{-p}{300-3p} \cdot (-3)$

$= \dfrac{3p}{300-3p} = \dfrac{p}{100-p}$

b) $E(80) = \dfrac{80}{100-80} = 4$

Demand is elastic. A 1% increase in price causes 4% decrease in demand.

③ $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

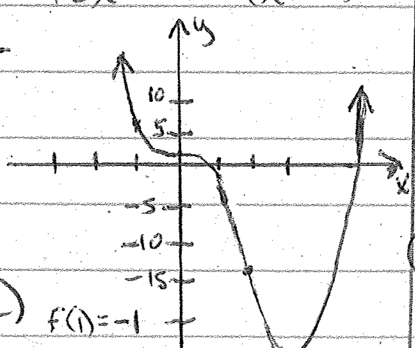
dec | dec | inc
0 3

levels off

$f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

c.u. | c.d. | c.u.
0 2



$f(1) = -1$

$f(0) = 2$, $f(2) = -14$

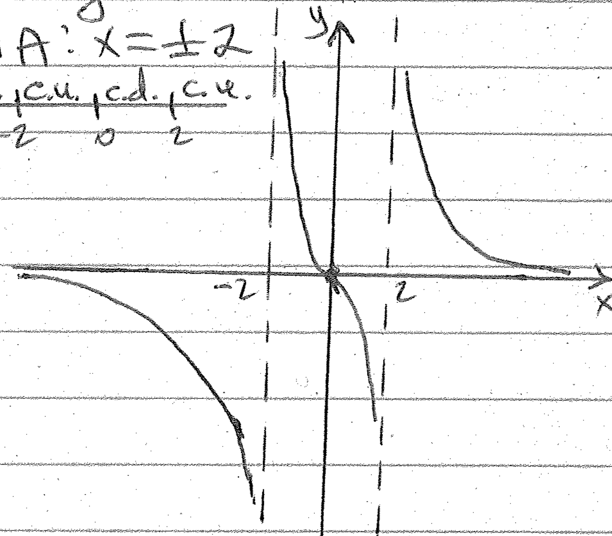
$f(-1) = 7$, $f(3) = -25$

④ $f' < 0 \Rightarrow f$ dec. (on each branch)

HA: $y = 0$ $\quad f(0) = 0$

VA: $x = \pm 2$

c.d. | c.u. | c.d. | c.u.
-2 0 2



⑤ $f'(x) = 6x^2 - 14x + 8$

$= 2(3x-4)(x-1) = 0$

$x = \frac{4}{3}, x = 1$

$f(0) = 2^{\text{min}}, f(1) = 5, f(\frac{4}{3}) = 4\frac{26}{27}$

$f(3) = 17 \leftarrow \text{max}$

⑥ a) $A(q) = \dfrac{3q^2 + 3q + 108}{q}$

$A(q) = 3q + 3 + 108q^{-1}$

$A'(q) = 3 - 108q^{-2} = 0$

$3 = \dfrac{108}{q^2} \Rightarrow q^2 = 36 \Rightarrow q = 6$

b) $3q + \cancel{3} + 108q^{-1} = 6q + \cancel{3}$

$108q^{-1} = 3q$

$36 = q^2 \Rightarrow q = 6$

⑦ $R'(p) = 300 - 3p^2 = 0$

$3p^2 = 300 \Rightarrow p^2 = 100 \Rightarrow p = 10$

$R_{max} = 300(10) - 10^3 = 2{,}000$