No calculators allowed.

(65 pts.)

MAC 2233 EXAM V MR. NADEL SUMMER 2017

(20) ① a) Find the future value of a continuous income stream with a flow rate of \$1,100 per year at 7% compounded continuously for 5 years.

b) If $p = D(q) = \frac{2000}{7q+80}$ $\frac{\text{dollars}}{\text{unit}}$, find the total amount of money consumers are willing to spend to get 10 units. ($q=10$)

c) Now, find the actual consumers' expenditure in b).

d) When a machine is $t$ years old, $R'(t) = 400 - t^2$ $\frac{\text{dollars}}{\text{year}}$ and $C'(t) = t^2 + 20t$ $\frac{\text{dollars}}{\text{year}}$. Find the useful life of the machine.

(15) ② $f(x,y) = \frac{2x-4y}{x+y}$

$g(x,y) = \frac{x-y}{e^{xy}}$

a) Find domain of $f$.

b) Find domain of $g$.

c) Compute $f(1,3)$.

(15) ③ a) If $f(x,y) = xe^{xy}$ find $f_y$ and $f_x$.

b) If $f(x,y) = x\ln(1+2x-5y)$ find $f_{yy}$

③ c) If weekly output is $Q(x,y) = 12x + 50y - 2x^2 + y^2$ units, use marginal analysis to estimate the change in weekly output, if $x$ increases from 20 to 21, while $y$ remains constant at 10.

(15) ④ Suppose a cost function is

$C(x,y) = 2x^2 + 2xy + 3y^2 - 16x - 18y + 54$

where $x$ = amount spent on labor, $y$ = amount spent on equipment.

Determine how much should be spent on each to minimize the cost. Also find the minimum cost. (Units are dollars.) Justify, using the Second Partials Test, and show all work.

(Calculator answers provided as a courtesy.)

MAC 2233 EXAM V KEY (SU '17)

① a) $e^{.07(5)}\int_0^5 1100e^{-.07t}\,dt$

$= \frac{e^{.35}(1100)}{-.07} e^{-.07t}\Big|_0^5$

$= \frac{e^{.35}(1100)}{-.07}(e^{-.35}-1)$

$= \$6585.35$

b) $\int_0^{10} \frac{2000}{7q+80}\,dq$

$= \frac{2000}{7}\ln|7q+80|\Big|_0^{10}$

$= \frac{2000}{7}(\ln 150 - \ln 80) = \$179.60$

c) $D(10) = \frac{2000}{150}$

Then $\frac{2000}{150}(10) = \frac{2000}{15} = \$133.33$

d) $400 - t^2 = t^2 + 20t$

$0 = 2t^2 + 20t - 400$

$0 = t^2 + 10t - 200 = (t+20)(t-10)$

$t = 10,\ t = \cancel{-20}$

② a) $x + y \neq 0$ OR $y \neq -x$

b) all $(x, y)$ $x, y$ are real nos.

c) $\frac{2-12}{1+3} = \frac{-10}{4} = \frac{-5}{2}$

③ a) $f_y = xe^{xy}\cdot x = x^2e^{xy}$ (C.R.)

$f_x = xe^{xy}\cdot y + e^{xy}(1)$ by P.R. (C.R.)

③ b) $f_y = x\frac{1}{1+2x-5y}(-5)$ (C.R.)

$= -5x(1+2x-5y)^{-1}$

$f_{yy} = 5x(1+2x-5y)^{-2}(-5)$ (C.R.)

c) $Q_x = 12 - 4x$

$12 - 4(20) = -68$

④ $C_x = 4x + 2y - 16 = 0$

$C_y = 2x + 6y - 18 = 0$

Solve system $\Rightarrow x = 3, y = 2$. (Show work)

$2x + y = 8$

$-2x - 6y = -18$

$-5y = -10$

$y = 2$

$2x + 2 = 8$

$2x = 6 \Rightarrow x = 3$

$C_{xx} = 4$

$C_{xy} = 2$

$C_{yy} = 6$

$D = C_{xx}C_{yy} - (C_{xy})^2$

$= 4(6) - 2^2 = 20 > 0.$

$C_{xx} > 0 \Rightarrow$ relative minimum

$C(3, 2) =$

$18 + 12 + 12 - 48 - 36 + 54 = 12$

is minimum cost.