

No calculators allowed.

MAC 2233 EXAM III MR. NADEL SUMMER 2017

(1) a) A grower estimates that if 50 orange trees are planted, the average yield per tree will be 300 oranges. The average yield will decrease by 3 oranges per tree for each additional tree planted on the same acreage.

How many trees should the grower plant to maximize total yield?

Let  $x$  = no. of additional trees and set up a closed interval.

b) An open box is built with a square base. The sides cost \$3 per sq. ft, and the base costs

\$4 per sq. ft. What are the dimensions of the box with greatest volume that can be made for \$48?

(2) a) If \$3,000 is invested at an annual rate of 7% for 5 years, find the balance if interest is compounded monthly?

b) Repeat a) but compounded continuously.

c) Sketch  $y = 3^x$

(3) a) How long will it take money to double if it's compounded continuously at a 5% annual rate?

b) Simplify  $\log_7(x^4 y^{-6})$

c) Solve for  $x$ :  $2 - 3 \ln x = 4$

(4) Find the derivatives:

a)  $y = \ln(x^2 - 3)^{200}$  using a property of logarithms.

b)  $y = 5x^2 e^{(x^2 - 3x + 9)}$

Use logarithmic differentiation on c) and d):

c)  $y = \frac{\sqrt[3]{7x^2 - 2}}{x^3 + 4}$

d)  $y = 4^x$

(5) a) Number of burgers sold at a chain is growing exponentially. If 3 billion were sold by 2005 and 12 billion by 2010, how many were sold by 2015?

b) Sketch

$$y = \frac{2}{1 + 3e^{2x}}$$

c) A worker can sort  $Q(t) = 800 - 500e^{-0.4t}$  letters/hr after  $t$  months on the job. How many can he ultimately sort?

MAC 2233 EXAM III KEY (SU'17)

① a)  $f(x) = (50+x)(300-3x)$

on  $0 \leq x \leq 100$

$f(0) = 15,000$   $f(100) = 0$

$f'(x) = (50+x)(-3) + (300-3x)(1) = 0$

$-150 - 3x + 300 - 3x = 0$

$6x = 150 \Rightarrow x = 25$  ← max

$f(25) = 75(225) = 16,875$

$50 + 25 = 75$  trees

b)  $4x^2 + 3(4xy) = 48$

$4x^2 + 12xy = 48 \Rightarrow x^2 + 3xy = 12$

$y = \frac{12-x^2}{3x} (x)$

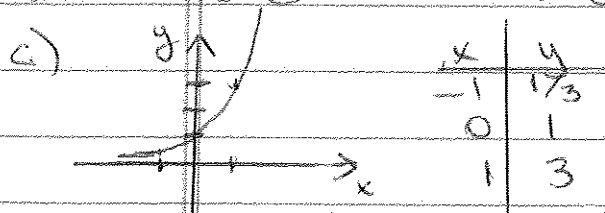
$V = x^2 y = x^2 \left( \frac{12-x^2}{3x} \right) = 4x - \frac{1}{3}x^3$

$V' = 4 - x^2 = 0 \Rightarrow x = 2$

$y = \frac{12-2^2}{3(2)} = \frac{8}{6} = \frac{4}{3}$

② a)  $B = P(1+i)^n = 3000 \left(1 + \frac{.07}{12}\right)^{60}$

b)  $3000 e^{.07(5)} = 3000 e^{.35}$



③ a)  $B = Pe^{rt}$ ,  $2P = Pe^{.05t}$

$\ln 2 = .05t$

$t = \frac{\ln 2}{.05}$

b)  $\log_7 x^4 + \log_7 y^{-6}$   
 $= 4 \log_7 x - 6 \log_7 y$

③ c)  $-3 \ln x = 2 \Rightarrow \ln x = -\frac{2}{3}$

$x = e^{-2/3}$

④ a)  $y = 200 \ln(x^2 - 3)$

$y' = 200 \left( \frac{1}{x^2-3} \right) (2x)$  ← C.R.

b)  $f'(x) = 5x^2(x^2-3x+9)(2x-3) + e^{(x^2-3x+9)}(2x)$  ← C.R.

by P.R.

c)  $\ln y = \frac{1}{3} \ln(7x^2-2) - \ln(x^3+4)$

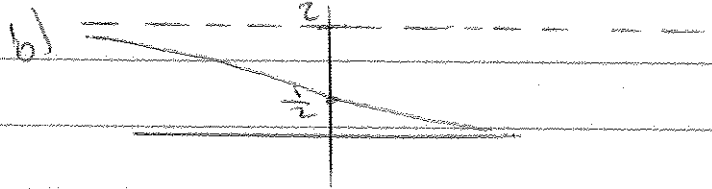
$\frac{y'}{y} = \left[ \frac{14x \leftarrow \text{C.R.}}{3(7x^2-2)} - \frac{\text{C.R.} \rightarrow 3x^2}{x^3+4} \right]$

$y' = y \cdot (\dots)$

d)  $y' = 4^x (\ln 4)$

⑤ a) quadruples every 5 years

$12(4) = 48$  billion



c)  $\lim_{t \rightarrow \infty} (800 - 500e^{-0.4t})$   
 $= 800 - 0 = 800$