No calculators allowed.

(75 pts.)

MAC 2233 EXAM I MR. NADEL SUMMER 2017

(10)① a) $\lim_{x\to-\infty} \frac{2x^2-3x+5}{9x^2+7}$

b) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

c) $\lim_{x\to 4} \frac{x^2-3x-4}{x^2-5x+4}$

d) $\lim_{x\to\infty} \frac{2x^2-3}{x^4+5}$

(10)② a) $\lim_{x\to 3^-} \frac{x+2}{x-3}$

b) Where is $f(x)=\frac{2x-3}{4x+2}$ not continuous?

c) Find the value of A so

$$f(x)=\begin{cases} 1-2x & x<3 \\ Ax^2+3x-1 & x\geq 3 \end{cases}$$

will be continuous for all x.

(10)③ Find the derivative of $y=f(x)=\frac{2}{x}$ using the definition of derivative. (the one with "h")

(5)④ If $h(t)=-\frac{1}{2}t^2+30t+20$ ft. is the height of a toy rocket after t seconds, find the instantaneous velocity after 25 seconds.

(10)⑤ a) Find an equation of the tangent line to $y=x^2+\frac{3}{x^2}$ at x=1. Don't use Q.R.

b) x yrs. after 2012, property tax was $T(x)=20x^2+40x+600$ dollars. At what rate was the tax increasing with respect to time in 2015?

(5)⑥ If $y=x^{-1}-2x^{-2}-3x^{-3}$ find y'' when x=1. (second derivative)

(25)⑦ a) If $y=u^3-3u+1$ and $u=x^4-7x^2$, find $\frac{dy}{dx}$ in terms of x.

b) Find y' without Q.R.:

$$y=\frac{2x}{(x^2+3x+4)^3}$$

c) Find y' if $y=\sqrt{x^4-3x^3+28x}$

d) Find y' if $y=\frac{(1-5x)^{30}}{(x^2+1)^{20}}$

e) Total cost of producing q units is $C(q)=.03q^2+q+800$ dollars. If $q(t)=t^2+90t$ units are produced after t hours, compute the rate at which the manufacturing cost is changing with respect to time 2 hours after production begins.

MAC 2233 EXAM I KEY (SU'17)

① a) $2/9$

b) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim_{x\to 4} \frac{(x-4)1}{(x-4)(\sqrt{x}+2)} = \frac{1}{2+2} = \frac{1}{4}$

c) $\lim_{x\to 4} \frac{(x-4)(x+1)}{(x-4)(x-1)} = \frac{5}{3}$

d) 0

② a) $-\infty$

b) $4x+2=0 \Rightarrow x=-\frac{1}{2}$

c) $1-2(3) = A(3^2)+9-1$

$-5 = 9A+8 \Rightarrow A = -\frac{13}{9}$

③ $\frac{f(x+h)-f(x)}{h} = \frac{\frac{2}{x+h}-\frac{2}{x}}{h}$

$= \frac{1}{h}\left(\frac{2}{x+h}-\frac{2}{x}\right)$

$= \frac{1}{h}\left[\frac{2x-2(x+h)}{x(x+h)}\right] = \frac{1}{h}\left[\frac{2x-2x-2h}{x(x+h)}\right]$

Let $h\to 0$, $f'(x) = \frac{-2}{x^2}$.

④ $v(t) = -t+30$

$v(25) = -25+30 = 5$ ft/sec

⑤ a) $y = x^2+3x^{-2}$ $(1,4)$

$y' = 2x-6x^{-3}$

$m_{tan} = 2-6 = -4$

$y-4 = -4(x-1)$

or $y = -4x+8$

b) $T'(x) = 40x+40$

$T'(3) = 40(3)+40 = 160$

⑥ $y' = -x^{-2}+4x^{-3}+9x^{-4}$

$y'' = 2x^{-3}-12x^{-4}-36x^{-5}$

At $x=1$, $y'' = 2-12-36 = -46$

⑦ a) $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= (3u^2-3)(4x^3-14x)$

$= \left[3(x^4-7x^2)^2-3\right](4x^3-14x)$

b) $y = 2x\cdot(x^2+3x+4)^{-3}$ C.R.

$y' = 2x(-3)(x^2+3x+4)^{-4}(2x+3)$

$+ (x^2+3x+4)^{-3}(2)$

by P.R.

c) $y = (x^4-3x^3+28x)^{1/2}$ C.R.

$y' = \frac{1}{2}(x^4-3x^3+28x)^{-1/2}(4x^3-9x^2+28)$

d) $\frac{(x^2+1)^{20}30(1-5x)^{29}(-5)-(1-5x)^{30}20(x^2+1)^{19}(2x)}{(x^2+1)^{40}}$

C.R. C.R.

e) $\frac{dC}{dt} = \frac{dC}{dq}\cdot\frac{dq}{dt}$

$t=2 \Rightarrow q = 4+180 = 184$

$\frac{dC}{dt} = (.06q+1)(2t+90)$

$= (.06(184)+1)(4+90)$

$= 1131.76$