# The Binomial Theorem

Recall that you can't distribute exponents over terms. For example,

$$(a+b)^2 \neq a^2 + b^2 \quad \text{and} \quad (a+b)^3 \neq a^3 + b^3 .$$

But $(a+b)^0 = 1$ , $(a+b)^1 = 1a + 1b$ , and $(a+b)^2 = a^2 + 2ab + b^2$ .

Also, $(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + b^3$ . And so on.

There is a pattern here. The powers on the "a" decrease. b comes into the second term, and the powers on it increase. The coefficient of the second term is the same as the original exponent. To get the coefficient of a term, multiply the coefficient of the previous term by the power of a, and divide by one more than the power on b.

For example, $(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$ .

The 6 is obtained by multiplying the previous 4 by 3, and dividing by 1 more than the exponent 1 on the b. In other words , 12 divided by 2.

Another way to obtain the coefficients is Pascal's triangle:

1<br>
1 1<br>
1 2 1<br>
1 3 3 1<br>
1 4 6 4 1<br>
1 5 10 10 5 1

Add any two numbers and write the answer below and between them. (Note the symmetric nature of the coefficients.

<u>Exercises:</u> 1) Write out the next 3 rows in Pascal's triangle.

2)Expand $(a+b)^5$

3)Expand $(a+b)^6$

In general, $(a+b)^n = a^n + na^{n-1}b + \dfrac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + b^n$ .